Intro. to ODEs

Quiz 6 Solutions

1) Consider the following matrices.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -1 \\ -1 & 5 \\ 3 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 0 \\ 1 & 1 & -2 \end{bmatrix}$$

If the products below exist, compute them. Otherwise, say that they are undefined.

a) AB

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 5 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 9 & 9 \\ 4 & 1 \end{bmatrix}$$

b) BA

$$\begin{bmatrix} 2 & -1 \\ -1 & 5 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 4 & 4 \\ -6 & -2 & 7 \\ 3 & 6 & 9 \end{bmatrix}$$

c) BC

This is undefined since B is 3×2 while C is 3×3. Note that CB would be a well-defined product for these two matrices!

d) $B^T C$

$$\begin{bmatrix} 2 & -1 & 3 \\ -1 & 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 0 \\ 1 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 4 & -4 \\ 9 & -5 & -1 \end{bmatrix}$$

2) Find the inverse of the following matrix by adjoining the identity matrix and reducing to RREF.

$$M = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 2 & 2 & -1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 2 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & -2 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -2 & -2 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 2 & -1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 2 & -1 \\ 0 & 1 & 1 & -2 & -1 & 1 \\ 0 & 0 & 1 & 2 & 2 & -1 \end{array}\right]$$

This gives us

$$M^{-1} = \begin{bmatrix} 3 & 2 & -1 \\ -2 & -1 & 1 \\ 2 & 2 & -1 \end{bmatrix}.$$